INTRODUCTION

Proofs: Involving Segments

GEOMETRIC PROOF 1

GEOMETRIC PROOF 2

Proofs: Involving Angle Relationships

GEOMETRIC PROOF 3

GEOMETRIC PROOF 4

GEOMETRIC PROOF 5

GEOMETRIC PROOF 6

GEOMETRIC PROOF 7

Standard 1:

Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

Standard 2:

Students write geometric proofs, including proofs by contradiction.

Standard 3:

Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.

Deductive Reasoning: Algebra

FORMAL

Two column proofs:

Given: $4(x + 2) = 2x + 18$

Prove: $x = 5$

Proof:

Statements	Reasons
(1) $4(x + 2) = 2x + 18$	(1) Given
(2) $4x + 8 = 2x + 18$	(2) Distributive prop.
(3) $4x = 2x + 10$	(3) Subtraction prop. (=)
(4) $2x = 10$	(4) Subtraction prop. (=)
(5) $x = 5$	(5) Division Prop. (=)

INFORMAL

$$4(x + 2) = 2x + 18$$

$$4x + 8 = 2x + 18$$

$$-8 \qquad -8$$

$$4x = 2x + 10$$

$$-2x \qquad -2x$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

Congruence in segments and angles is Reflexive, Symmetric and Transitive:

$\cong$ of segments is *reflexive*.

$\overline{LM} \cong \overline{LM}$

$\cong$ of segments is *symmetric*.

$\overline{KL} \cong \overline{LM}$ $\quad$ $\overline{LM} \cong \overline{KL}$

$\cong$ of segments is *transitive*.

$\overline{KL} \cong \overline{LM}$

$\overline{LM} \cong \overline{AB}$

$\overline{KL} \cong \overline{AB}$

$\cong$ of $\angle$s is *reflexive*

$\angle ECA \cong \angle ECA$

$\cong$ of $\angle$s is *symmetric*

$\angle BCE \cong \angle FGH$ $\quad$ $\angle FGH \cong \angle BCE$

$\cong$ of $\angle$s is *transitive*

$\angle BCE \cong \angle FGH$

$\angle FGH \cong \angle ECA$

$\angle BCE \cong \angle ECA$

For all segments and angles, their measures comply with these same properties.

STANDARDS 1,2,3

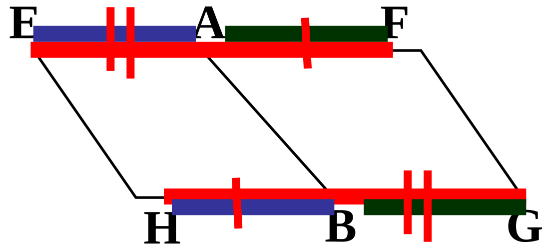
Given:

$\overline{EF} \cong \overline{HG}$

$\overline{HB} \cong \overline{AF}$

Prove:

$\overline{EA} \cong \overline{BG}$



Two Column Proof:

Statements	Reasons
(1) $\overline{EF} \cong \overline{HG}$	(1) Given
(2) $EF = HG$	(2) Definition of $\cong$ segments.
(3) $EF = EA + AF$ and $HG = HB + BG$	(3) Segment Addition Postulate.
(4) $EA + AF = HB + BG$	(4) Substitution prop. of (=)
(5) $\overline{HB} \cong \overline{AF}$	(5) Given
(6) $HB = AF$	(6) Definition of $\cong$ segments.
(7) $EA = BG$	(7) Subtraction prop. of (=)
(8) $\overline{EA} \cong \overline{BG}$	(8) Definition of $\cong$ segments.

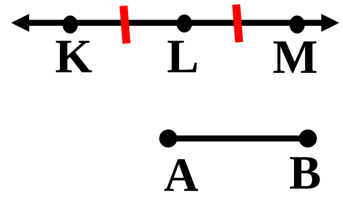
Given:

L is midpoint of $\overline{KM}$

$\overline{LM} \cong \overline{AB}$

Prove:

$\overline{KL} \cong \overline{AB}$



Two Column Proof:

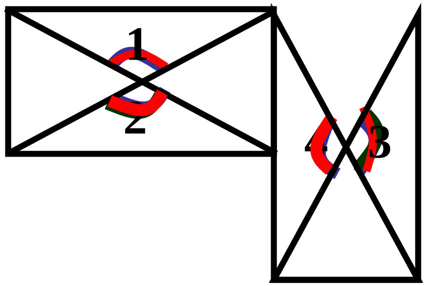
Statements	Reasons
(1) L is midpoint of $\overline{KM}$	(1) Given
(2) $\overline{KL} \cong \overline{LM}$	(2) Definition of Midpoint
(3) $\overline{LM} \cong \overline{AB}$	(3) Given
(4) $\overline{KL} \cong \overline{AB}$	(4) $\cong$ of segments is transitive.

Given:

$\angle 1 \cong \angle 3$

Prove:

$\angle 2 \cong \angle 4$



Two Column Proof:

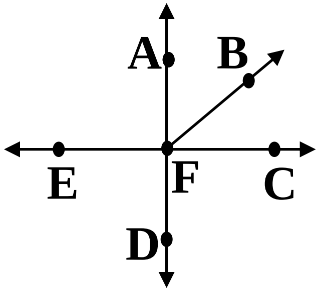
Statements	Reasons
(1) $\angle 1 \cong \angle 3$	(1) Given
(2) $\angle 1 \cong \angle 2$	(2) Vertical $\angle$S are $\cong$
(3) $\angle 2 \cong \angle 3$	(3) $\cong$ of $\angle$s is *transitive*
(4) $\angle 3 \cong \angle 4$	(4) Vertical $\angle$S are $\cong$
(5) $\angle 2 \cong \angle 4$	(5) $\cong$ of $\angle$s is *transitive*

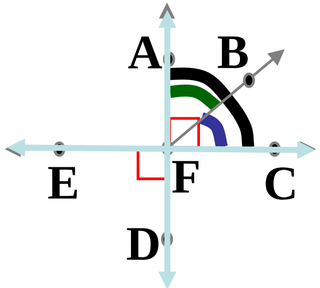
Given:

∠EFD is right

Prove:

∠AFB and ∠CFB are complementary.





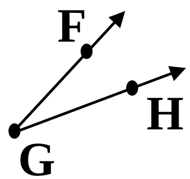
Two Column Proof:

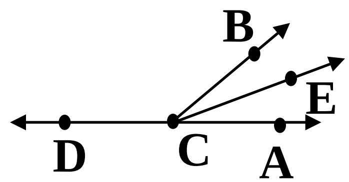
Statements	Reasons
(1) ∠EFD is right	(1) Given
(2) $\overleftrightarrow{EC} \perp \overleftrightarrow{AD}$	(2) Definition of ⊥ lines
(3) ∠AFC is right	(3) ⊥ lines form 4 right ∠s
(4) m∠AFC = 90°	(4) Definition of right ∠s
(5) m∠AFB + m∠CFB = m∠AFC	(5) ∠ addition postulate
(6) m∠AFB + m∠CFB = 90°	(6) Substitution prop. of (=)
(7) ∠AFB and ∠CFB are complementary.	(7) Definition of complementary ∠s

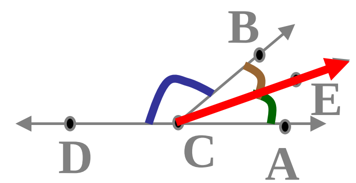
Given:

$\overrightarrow{CE}$ bisects ∠BCA

∠FGH ≅ ∠ECA







Prove:

2(m∠FGH) + m∠BCD = 180°

<u>Two Column Proof:</u>

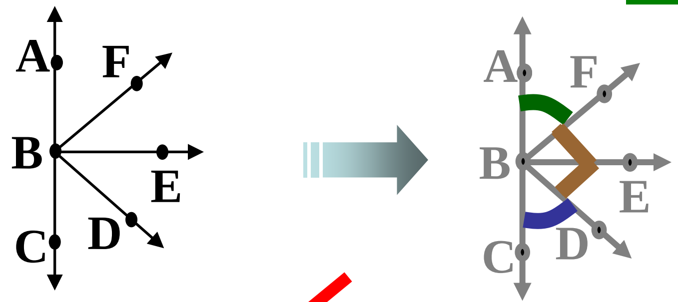
Statements	Reasons
(1) $\overrightarrow{CE}$ bisects ∠BCA	(1) Given
(2) ∠BCE ≅ ∠ECA	(2) Definition of ∠ bisector
(3) m∠BCE = m∠ECA	(3) Definition of ≅ ∠s
(4) ∠FGH ≅ ∠ECA	(4) Given
(5) m∠FGH = m∠ECA	(5) Definition of ≅ ∠s
(6) m∠BCE = m∠FGH	(6) ≅ of ∠s is transitive
(7) m∠ECA + m∠BCE + m∠BCD = 180°	(7) ∠ addition postulate
(8) m∠FGH + m∠FGH + m∠BCD = 180°	(8) Substitution prop. of (=)
(9) 2(m∠FGH) + m∠BCD = 180°	(9) Adding like terms

Given:

$\angle$FBD is right

Prove:

$\angle$ABF and $\angle$CBD are complementary.



Two Column Proof:

Statements	Reasons
(1) $\angle$FBD is right	(1) Given
(2) $m\angle FBD = 90°$	(2) Definition of right $\angle$s
(3) $m\angle ABF + m\angle FBD + m\angle CBD = 180°$	(3) $\angle$ addition postulate
(4) $m\angle ABF + 90° + m\angle CBD = 180°$	(4) Substitution prop. of (=)
(5) $m\angle ABF + m\angle CBD = 90°$	(5) Subtraction prop. of (=)
(6) $\angle$ABF and $\angle$CBD are complementary.	(6) Definition of complementary $\angle$s

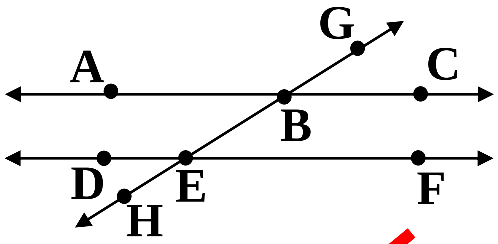
Given:

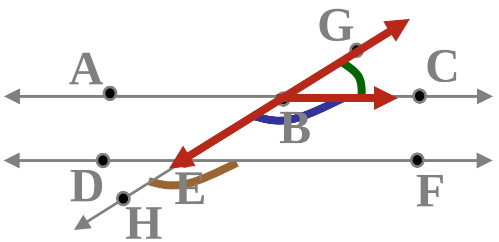
$\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ are //

$\overleftrightarrow{GE}$ is a transversal

Prove:

∠GBC and ∠FEH are supplementary.





Two Column Proof:

Statements	Reasons
(1) $\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ are // $\overleftrightarrow{GE}$ is a transversal	(1) Given
(2) ∠GBC and ∠CBE are a linear pair	(2) Definition of linear pair
(3) m∠GBC + m∠CBE = 180°	(3) ∠s in a linear pair are supplementary
(4) ∠CBE ≅ ∠FEH	(4) In // lines cut by a transversal CORRESPONDING ∠s are ≅
(5) m∠CBE = m∠FEH	(5) Definition of ≅ ∠s
(6) m∠GBC + m∠FEH = 180°	(6) Substitution prop. of (=)
(7) ∠GBC and ∠FEH are supplementary.	(7) Definition of supplementary ∠s