Basic Polygon Definitions

Interior Angle Sum Theorem

PROBLEM 1a

PROBLEM 1b

PROBLEM 2a

PROBLEM 2b

Exterior Angle Sum Theorem

PROBLEM 3a

PROBLEM 3b

PROBLEM 4a

PROBLEM 4b

PROBLEM 5a

PROBLEM 5b

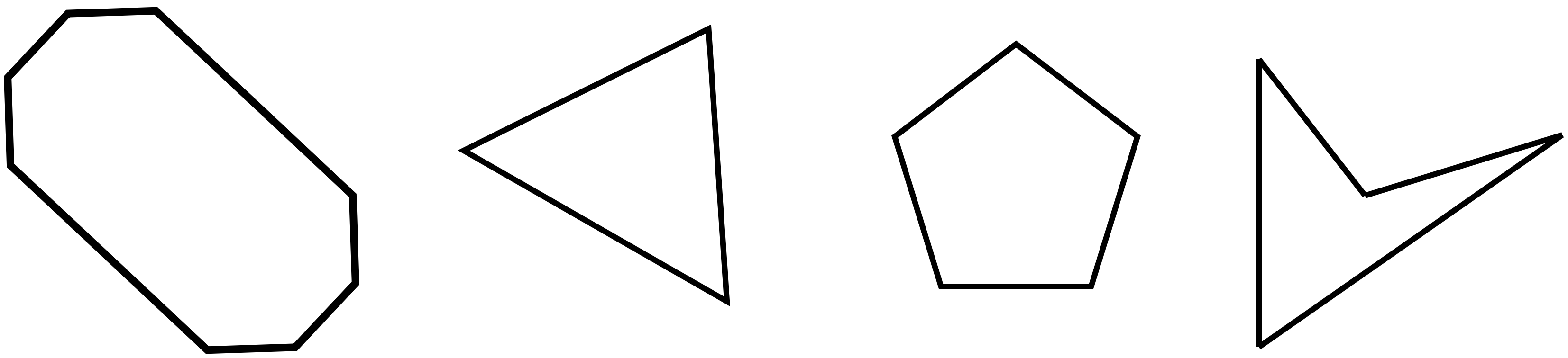
Standard 12:

Students find and use measures of sides, interior and exterior angles of triangles and polygons to classify figures and solve problems.

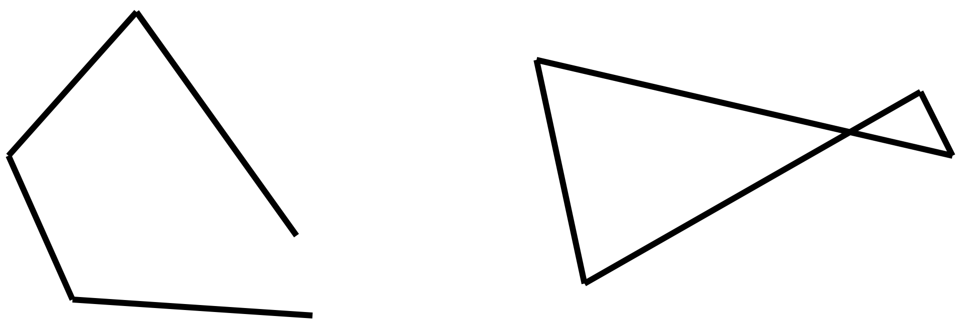
Estándar 12:

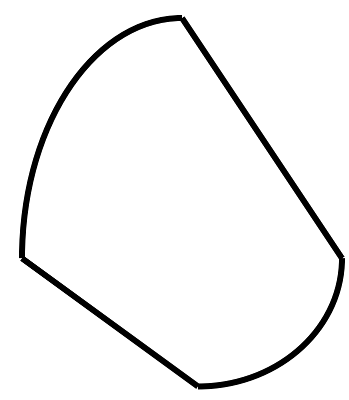
Los estudiantes encuentran y usan medidas de los lados, ángulos interiores y exteriores de triángulos y polígonos para clasificar figuras y resolver problemas.

These are examples of POLYGONS:



These are NOT POLYGONS:





What is the difference? Click to find out…

***Definition of a POLYGON*:**

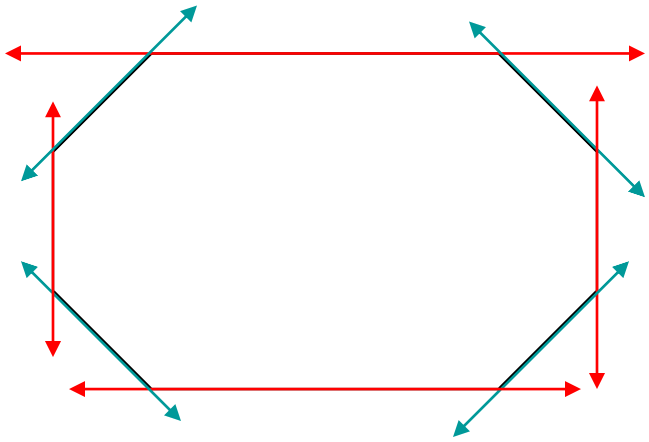
A polygon is closed figure formed by a finite number of coplanar segments such that

1.- the sides that have a common endpoint are noncollinear, and

2.- each side intersects exactly two other sides, but only at their endpoints.

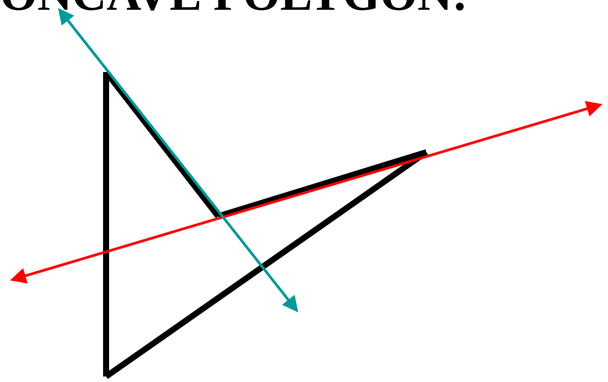
Do you know what is a Convex Polygon? Click to find out…

CONVEX POLYGON:



None of the lines containing a side of the polygon contains a point inside the polygon.

NON-CONVEX POLYGON OR CONCAVE POLYGON:

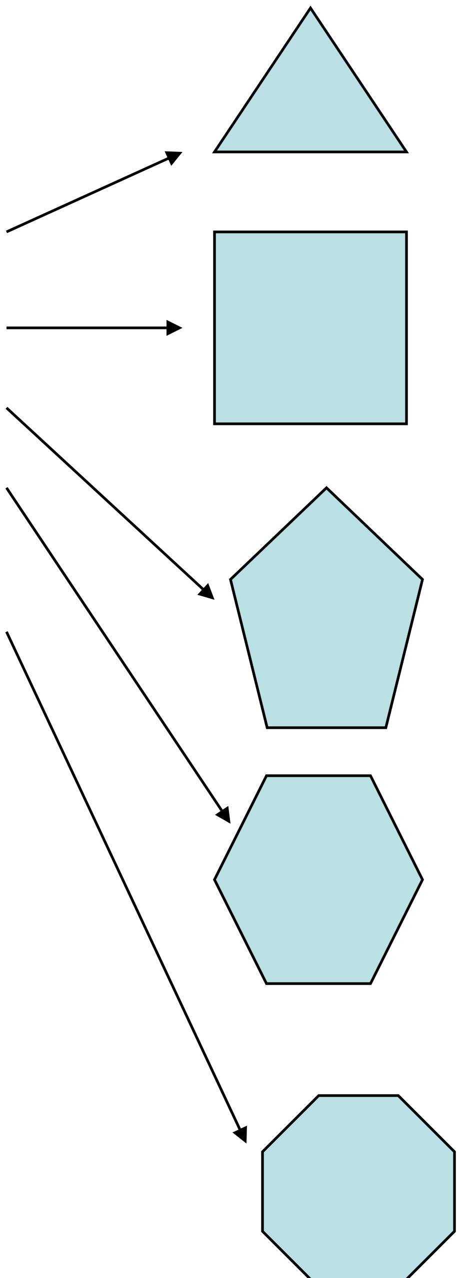


One or more lines containing a side on the polygon, contain points inside the polygon.

How do we call a polygon with 10 sides? Click to find out…

Standard 12

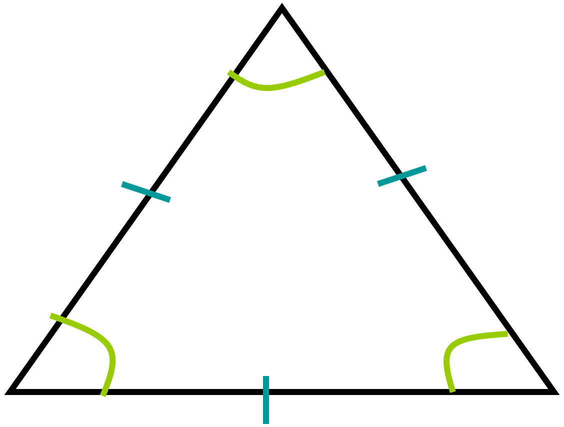
Number of sides	Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
N	ngon

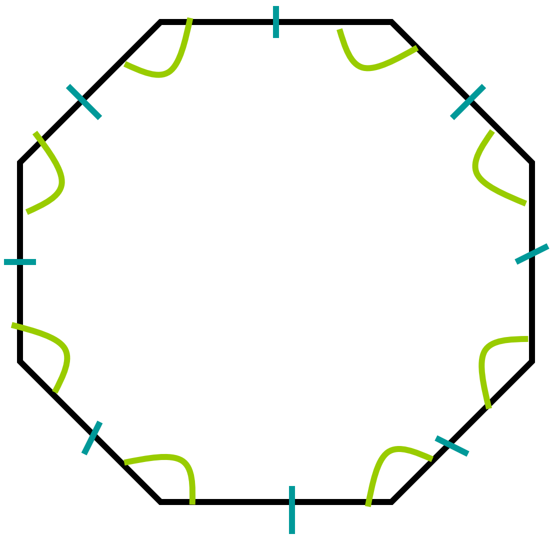


What is a regular polygon? Click to find out…

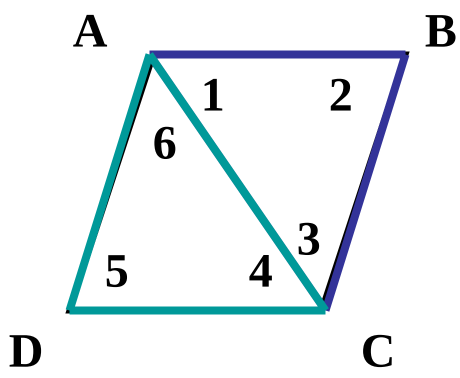
***Definition of Regular Polygon*:**

A regular polygon is a convex polygon with all sides congruent and all angles congruent.





What is the sum of the interior angles in the quadrilateral below?



We draw diagonal $\overline{AC}$

then we have $\triangle ABC$ and $\triangle ADC$

numbering all angles in $\triangle ABC$

numbering all angles in $\triangle ADC$

$$m\angle 1 + m\angle 2 + m\angle 3 = 180°$$

then

$$m\angle 4 + m\angle 5 + m\angle 6 = 180°$$

Because the sum of the interior angles of a triangle is 180°

adding both equations

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 180° + 180°$$

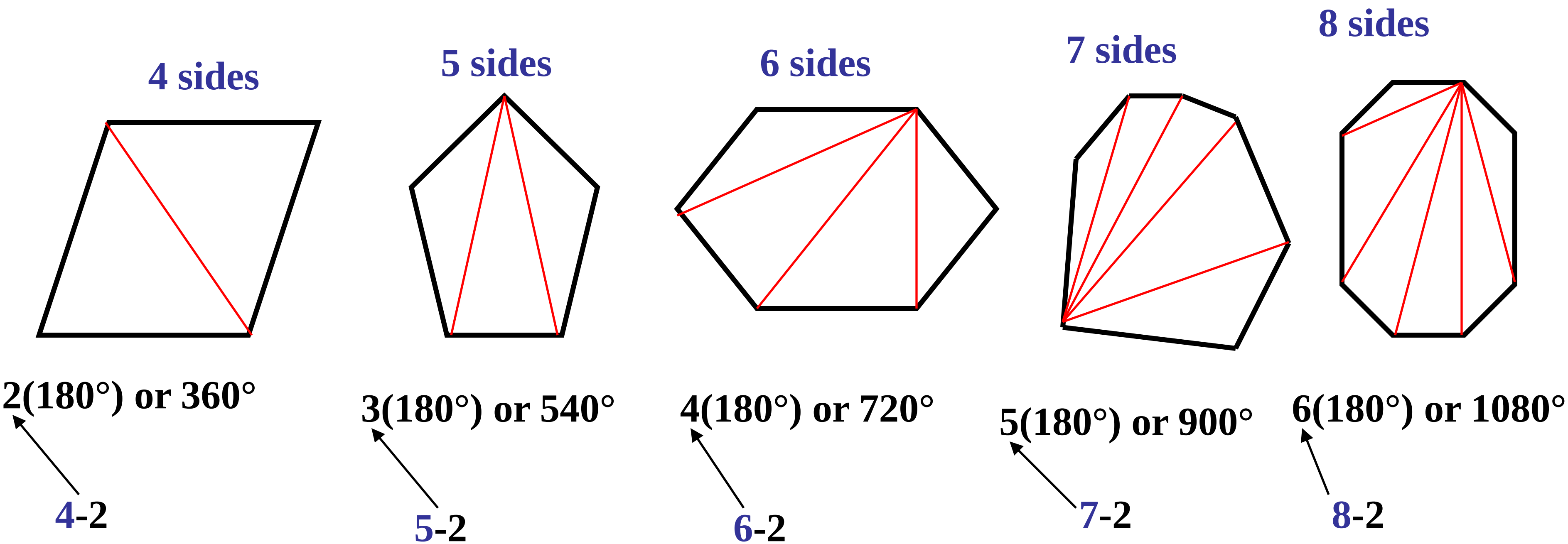
Rearrenging the terms

$$\underbrace{m\angle 1 + m\angle 6}_{m\angle A} + \underbrace{m\angle 2}_{m\angle B} + \underbrace{m\angle 3 + m\angle 4}_{m\angle C} + \underbrace{m\angle 5}_{m\angle D} = 180° + 180°$$

$$m\angle A + m\angle B + m\angle C + m\angle D = 2(180°) \text{ or } 360°$$

Could this be done for a Pentagon, an hexagon, etc.? Click to find out…

What would be the sum of the interior angles for all these polygons?



Can you see a pattern? How this relates to the number of sides?

(number of sides-2)180°=Sum of interior angles in the polygon

How can we define this? Click to find out…

Interior angle sum theorem

If a convex polygon has n sides and S is the sum of the measures of the interior angles, then:

$S=180°(n-2)$

Find the sum of the measures of the interior angles of a convex octagon.

We know that a Octagon is a polygon with 8 sides, so n=8

and $S=180°(n-2)$ **Where S= Sum of measures of interior angles**

So:

$S=180°(8-2)$

$S=180°(6)$

$$\boxed{S=1080°}$$

Find the sum of the measures of the interior angles of a convex decagon.

We know that a <u>Decagon</u> is a polygon with 10 sides, so n=10

and $S=180°(n-2)$ **Where S= Sum of measures of interior angles**

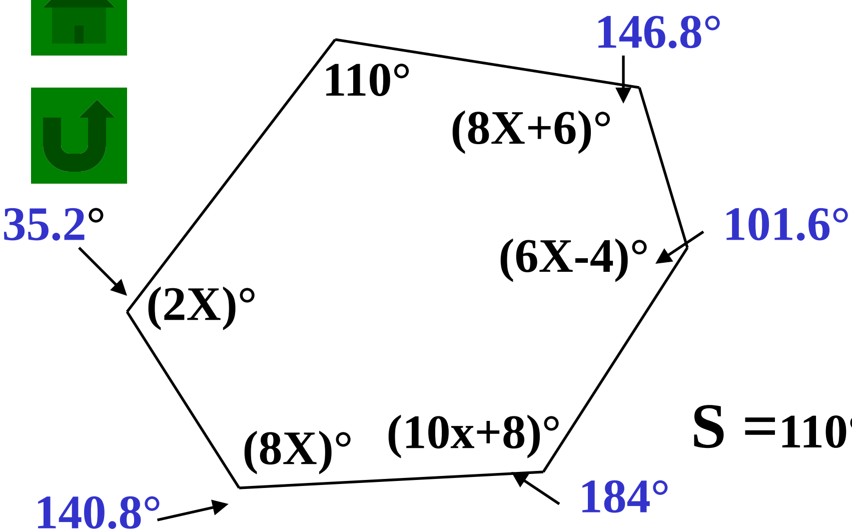
So:

$S=180°(10-2)$

$S=180°(8)$

$S=1440°$

Find the measure of each angle:



We know from the Interior Angle Sum Theorem that:

$$S=180°(n-2)$$

There are 6 sides so: n=6

and

$$S = 110° + (2X)° + (8X)° + (10X+8)° + (6X-4)° + (8X+6)°$$

So: $110° + (2X)° + (8X)° + (10X+8)° + (6X-4)° + (8X+6)° = 180°(6-2)$

$2X + 8X + 10X + 6X + 8X + 110 + 8 - 4 + 6 = 180(4)$

$34X + 120 = 720$

$-120 \quad -120$

$$\frac{34X}{34} = \frac{600}{34}$$

$X \approx 17.6$

Then:

$8X+6 = 8(17.6)+6 \approx$ **146.8°**

$6X-4 = 6(17.6)-4 \approx$ **101.6°**

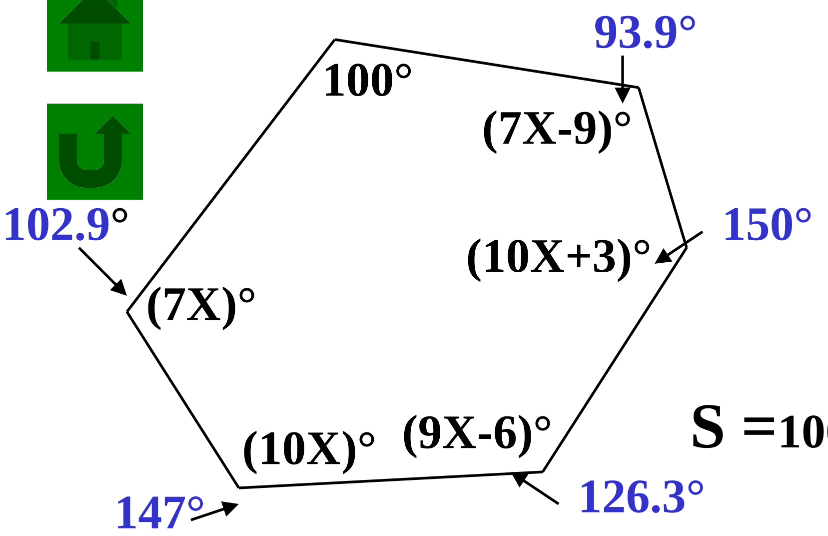
$10X+8 = 10(17.6)+8 \approx$ **184°**

$8X = 8(17.6) \approx$ **140.8°**

$2X = 2(17.6) \approx$ **35.2°**

Checking: 146.8° + 101.6°+184°+140.8°+35.2°+110° ≈ 720°

Find the measure of each angle:



We know from the Interior Angle Sum Theorem that:

$$S = 180°(n-2)$$

There are 6 sides so: n=6

and

$$S = 100° + (7X)° + (10X)° + (9X-6)° + (10X+3)° + (7X-9)°$$

So: $100° + (7X)° + (10X)° + (9X-6)° + (10X+3)° + (7X-9) = 180°(6-2)$

$7X + 10X + 9X + 10X + 7X + 100 - 6 + 3 - 9 = 180(4)$

$43X + 88 = 720$

$-88 \quad -88$

$\frac{43X}{43} = \frac{632}{43}$

$X \approx 14.7$

Then:

$7X-9 = 7(14.7)-9 \approx$ **93.9°**

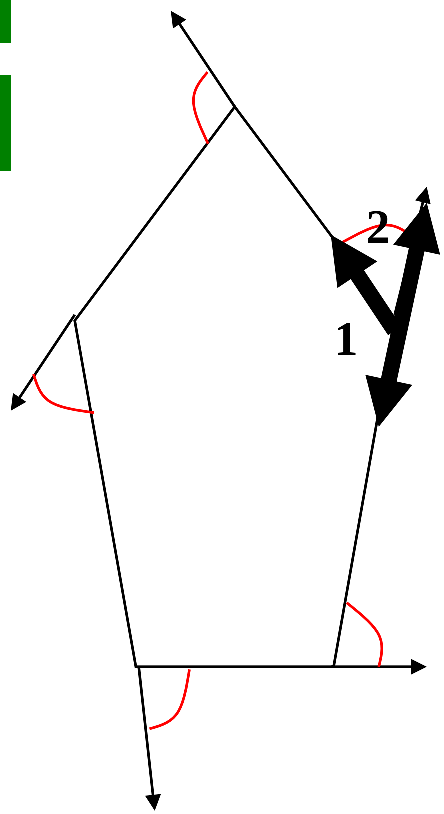
$10X+3 = 10(14.7)+3 \approx$ **150°**

$9X-6 = 9(14.7)-6 \approx$ **126.3°**

$10X = 10(14.7) \approx$ **147°**

$7X = 7(14.7) \approx$ **102.9°**

Checking: 93.9° + 150°+126.3°+147°+102.9°+100° ≈ 720°



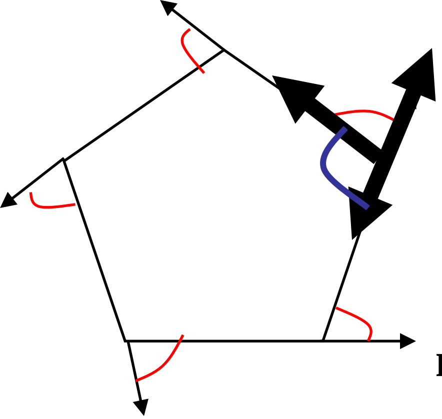
These are all the EXTERIOR ANGLES for the polygon at the right.

Angles $\angle 1$ and $\angle 2$ form a linear pair

$$m\angle 1 + m\angle 2 = 180°$$

Each exterior angle is supplementry to its interior angle.

What is the sum of the exterior angles in a polygon if we know the sum of the interior angles? Click to find out



Standard 12

For each vertex we have a linear pair, so:

measure of EXTERIOR ANGLE + measure of INTERIOR ANGLE =180°

substracting measure of INTERIOR ANGLE from both sides:

measure of EXTERIOR ANGLE=180°-measure of INTERIOR ANGLE

Multiplying both sides for n=number of vertices:

n(measure of EXTERIOR ANGLE)=n(180°-measure of INTERIOR ANGLE)

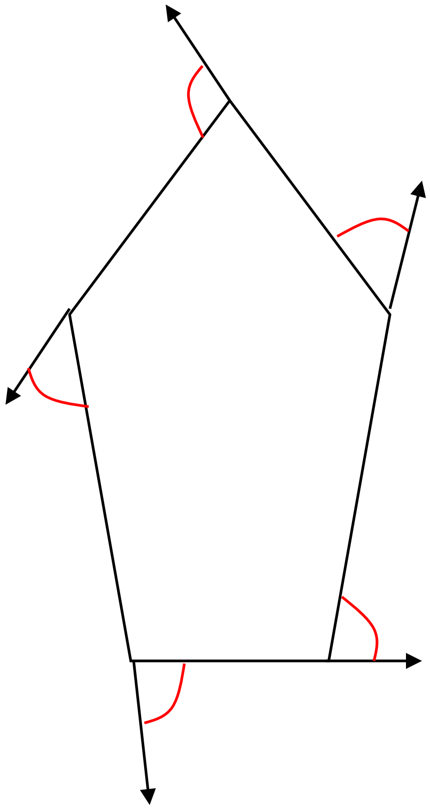
n(measure of EXTERIOR ANGLE)=n180°-n(measure of INTERIOR ANGLE)

EXTERIOR ANGLE SUM = n180° - INTERIOR ANGLE SUM

EXTERIOR ANGLE SUM = n180° - 180°(n-2)

EXTERIOR ANGLE SUM = n180°-n180°+360°

EXTERIOR ANGLE SUM= 360° **Click for a formal definition…**



<u>Exterior Angle Sum Theorem:</u>

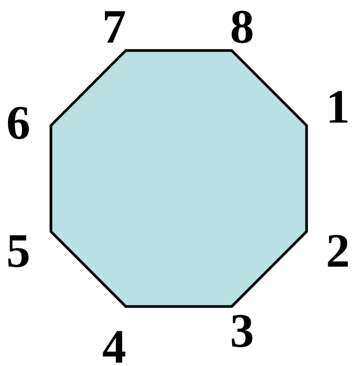
If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360°.

An exterior angle of a regular polygon is 45°. Find the number of sides.

From the Exterior Angle Sum Theorem, we know that the sum of the exterior angles is 360°, so:

$$\frac{360°}{45°} = 8$$

So we have 8 vertices, and <u>8 Sides.</u>

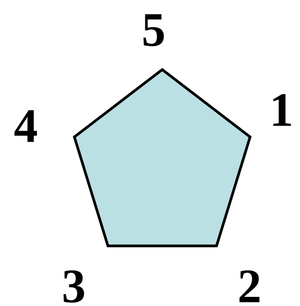


An exterior angle of a regular polygon is 72°. Find the number of sides.

From the Exterior Angle Sum Theorem, we know that the sum of the exterior angles is 360°, so:

$$\frac{360°}{72°} = 5$$

So we have 5 vertices, and <u>5 Sides.</u>



Find the number of sides in a regular polygon if the measure of each interior angle is 165°.

We know that in a vertex in the polygon, the interior angle forms a linear pair with the exterior angle. So:

measure of EXTERIOR ANGLE + measure of INTERIOR ANGLE =180°

substracting measure of INTERIOR ANGLE from both sides:

measure of EXTERIOR ANGLE=180°-measure of INTERIOR ANGLE

=180°-165°

=15°

And from Exterior Angle Sum Theorem, the sum of all exterior angles is 360°, so:

$$\frac{360°}{15°} = 24$$

There are 24 vertices and thus 24 sides.

Find the number of sides in a regular polygon if the measure of each interior angle is 156°.

We know that in a vertex in the polygon, the interior angle forms a linear pair with the exterior angle. So:

measure of EXTERIOR ANGLE + measure of INTERIOR ANGLE =180°

substracting measure of INTERIOR ANGLE from both sides:

measure of EXTERIOR ANGLE=180°-measure of INTERIOR ANGLE

=180°-156°

=24°

And from Exterior Angle Sum Theorem, the sum of all exterior angles is 360°, so:

$$\frac{360°}{24°}=15$$

There are 15 vertices and thus 15 sides.

Find the measures of each interior and each exterior angle of a regular 60-gon?

First we know that in a regular n-gon all interior and exterior angles are congruent and the sum of exterior angles is 360°, so:

$$\frac{360°}{60 \text{ vertices}} = \text{measure of EXTERIOR ANGLE}$$

$$\frac{360°}{60} = 6°$$

So: measure of EXTERIOR ANGLE = 6°

And the interior and exterior angles form a linear pair, so:

measure of EXTERIOR ANGLE + measure of INTERIOR ANGLE =180°

substracting measure of EXTERIOR ANGLE from both sides:

measure of INTERIOR ANGLE=180°-measure of EXTERIOR ANGLE

=180°-6°

measure of INTERIOR ANGLE=174°

Standard 12

Find the measures of each interior and each exterior angle of a regular 24-gon?

First we know that in a regular n-gon all interior and exterior angles are congruent and the sum of exterior angles is 360°, so:

$$\frac{360°}{24 \text{ vertices}} = \text{measure of EXTERIOR ANGLE}$$

$$\frac{360°}{24} = 15°$$

So: measure of EXTERIOR ANGLE = 15°

And the interior and exterior angles form a linear pair, so:

measure of EXTERIOR ANGLE + measure of INTERIOR ANGLE =180°

substracting measure of EXTERIOR ANGLE from both sides:

measure of INTERIOR ANGLE=180°-measure of EXTERIOR ANGLE

=180°-15°

measure of INTERIOR ANGLE=165°

Standard 12