

Big Picture

The perimeter and area of geometric shapes are basic properties that we need to know. The more complex a shape is, the more complex the process can be in finding its perimeter and area.

Key Terms

Perimeter: The distance around a shape.

Circumference: The distance around a circle.

Area: The amount of surface covered by a figure.

Center (of the polygon): The center of the circumscribed circle.

Radius (of the polygon): The radius of the circumscribed circle.

Apothem: A perpendicular segment from the center to a side of the polygon.

Units

Perimeter

The **perimeter** is the sum of all the edges of a two-dimensional figure. The perimeter is measured in units of length (e.g. feet, inches, centimeter). If the unit is not specified, the perimeter is measured in "units."

Circumference

The **circumference** is also measured in units of length (e.g. feet, inches, centimeter). If the unit is not specified, the circumference is measured in "units."

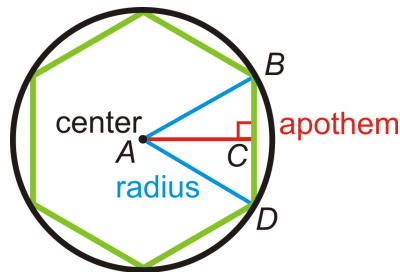
Area

The **area** is measured in square units (e.g. square feet or ft^2). If the unit is not specified, the area is measured in "units²."

Regular Polygons

A regular polygon is a polygon that is equiangular (all angles are equal) and equilateral (all sides have the same side lengths). All regular polygons can be inscribed in a circle, so these polygons also have a **center** and a **radius**.

- The radius for a regular polygon is the same as the radius of the circumscribed circle.
 - When the regular polygon is inscribed in a unit circle, the radius is 1.
- A central angle is the angle formed by two radii drawn to consecutive vertices of the polygon.
 - The angle measure is $\left(\frac{360^\circ}{n}\right)$
- Length of an **apothem** = $r \cos\left(\frac{180^\circ}{n}\right)$
 - r is the length of the radius
 - n is the number of sides



Perimeter of a Regular Polygon

The simplest way to find the perimeter of a regular polygon would be to just add the lengths of all the sides.

$$P = ns$$

- n = number of sides
- s = side lengths

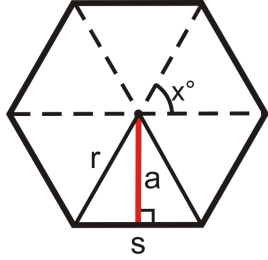
There is another version of the perimeter formula: $P = 2nr \sin\left(\frac{180^\circ}{n}\right)$

- n = number of sides
- r = radius length

PERIMETER AND AREA CONT.

Regular Polygons (cont.)

Area of a Regular Polygon



Theorem: $\text{Area} = \frac{1}{2}nas = \frac{1}{2}Pa$

- P = perimeter
- a = apothem
- The area of the triangle is $\frac{1}{2}as$, so the area of the regular polygon, which has n triangles, is n times the area of the triangle.

Can also be written as: $\text{Area} = nr^2 \sin\left(\frac{180^\circ}{n}\right) \cos\left(\frac{180^\circ}{n}\right)$

- n = number of sides
- r = radius length

Properties of Area and Perimeter

Congruent Areas Postulate: If two figures are congruent, they have the same area.

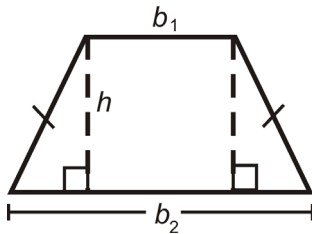
- The converse is not true! Two figures with the same area do not have to be congruent.

If the polygon is not a regular polygon, the area can be found by dividing the polygon into smaller polygons where the areas can be calculated.

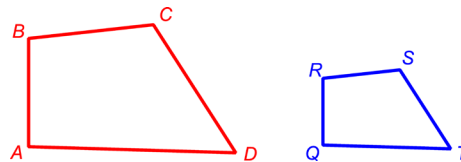
Area Addition Postulate: If a figure is composed of two or more parts that do not overlap each other, then the area of the figure is the sum of the areas of the parts.



If you remember the formulas for perimeter and area for rectangles and triangles, you can always divide other shapes into rectangles and triangles.



Perimeter of Similar Polygons Theorem: If two polygons are similar, then the ratio of the perimeters is equal to the ratio of the corresponding side lengths.



If $ABCD \sim QRST$, then $\frac{AB + BC + CD + DA}{QR + RS + ST + TQ} = \frac{AB}{QR} = \frac{BC}{RS} = \frac{CD}{ST} = \frac{DA}{TQ}$.

Area of Similar Polygons Theorem: If the scale factor of the sides of two similar polygons is $\frac{m}{n}$, then the ratio of the areas would be $\left(\frac{m}{n}\right)^2$.

If $ABCD \sim QRST$ and $\frac{AB}{QR}$ is the scale factor, then the ratio of the areas is $\left(\frac{AB}{QR}\right)^2$.